

A scheme for radiative CP violation

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Abstract

We present a simple model in which CP symmetry is spontaneously broken only after the radiative corrections are taken into account. The model includes two Higgs-boson doublets and two right-handed singlet neutrinos which induce the necessary non-hermitian interaction. To evade the Georgi-Pais theorem, some fine-tuning of coupling constants is necessary. However, we show that such fine-tuning is natural in the technical sense as it is protected by symmetry. Some phenomenological consequences are also discussed.

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More than thirty years after its experimental discovery, the origin of CP violation still remains very much a mystery. In the widely accepted Standard Model and many other models, CP violation is a result of the complex parameters[1] allowed in the Lagrangian. For many physicists, such mundane explanation of the origin of the violation of CP symmetry is not very satisfactory. In an effort to understand it at a deeper level, many different schemes have been conceived in the literature. A popular alternative is to require CP symmetry at the Lagrangian level and allow its nonconservation only in the vacuum. Such scheme are commonly termed spontaneous CP violation[2]. Another even more ambitious attempt is to consider CP as an exact symmetry at the tree level but allow its nonconservation only when the quantum effects are included. To realize such scheme within the perturbative framework, one naturally requires the CP violation to be spontaneous in origin also. Therefore, the model would have a Higgs potential in which its tree level ground states include a CP conserving one but when the radiative corrections are included, a CP violating ground state is selected[3]. In that case, one can genuinely call the CP violation a quantum mechanical effect. It is the aim of this paper to look for a realistic model of such type. Through out this paper, the CP symmetry is assumed to be an exact symmetry of the Lagrangian.

To implement this mechanism in any realistic model, there are two main obstacles. The first one is the Georgi-Pais theorem[4]. The theorem assumes that no fine-tuning of any kind is allowed. Under such assumption, the first conclusion one can make is that radiative CP violation is possible only if the degeneracy of the ground states of the tree level Higgs potential is such that CP symmetry cannot be asserted. That is, the CP violating ground states and the CP conserving ones are degenerate. Furthermore, Georgi and Pais also proved that radiative breaking can occur only if the tree level spectrum of the Higgs bosons contain a massless particle. Such boson may eventually pick up masses when the radiative effects are included. However, under the assumption of no fine-tuning such boson is necessarily light. Since the experimental limit on such light boson is very strong, it seems very difficult to find a realistic model under such scheme.

After the result of Georgi and Pais, there are many attempts to get around the constraint from the theorem. One can try to go beyond the perturbative framework [5] which is beyond our present scope. Alternatively, one can relax the no-fine-tuning constraint

and permit some fine-tuning as long as it is technically natural. (By technically natural, we mean, in this paper, a set of parameters can be assumed to be much smaller than the rest of the parameters as long as all the radiative corrections to these small parameters naturally contains powers of their small tree level values.) However, even if the Georgi-Pais theorem is circumvented by technically natural fine-tuning, its physical origin can still present itself in the form of the existence of a light Higgs boson in such model.

An example of such situation appears in the model proposed by Maekawa[6]. In the minimal supersymmetric model, it is well-known that spontaneous CP violation cannot happen at tree level. However, Maekawa showed that, if some parameters in the Higgs potential are much smaller than the gauge coupling, it is possible to have spontaneous CP violation when one-loop effect is taken into account. Following Maekawa's, Pomarol[7] pointed out that the Higgs boson spectrum of such model contains a light boson whose mass lies in the range that has already been ruled out by the LEP data[8]. In general the experimental bound on a pseudoscalar boson is not very strong because a pseudoscalar boson does not couple linearly to the Z boson directly. However, in the minimal supersymmetric model, this bound becomes more severe as the pseudoscalar boson mass can be related to the scalar boson mass.

Here we wish to present a simple Peccei-Quinn-type extension[9] of Standard Model in which the tree level vacuum is automatically CP symmetric and the radiative corrections induced by some of the Yukawa couplings can produce a CP violating vacuum. The idea here is *not* to champion a particular model, but to show that the fundamental mechanism underlying Ref.[6] has nothing to do with supersymmetry, and the problem[7] of a very light Higgs boson facing the model in Ref.[6] is also not intrinsic to the mechanism itself.

As in Peccei-Quinn[9] or supersymmetric[10] models, we start with two Higgs-boson doublets. In general, the following non-hermitian terms would appear in the Higgs potential,

$$-\mathcal{L}_{\text{Higgs}} = -m_{12}^2 \phi_1^\dagger \phi_2 + \lambda_5 (\phi_1^\dagger \phi_2)^2 + \lambda_6 \phi_1^\dagger \phi_1 \phi_1^\dagger \phi_2 + \lambda_7 \phi_1^\dagger \phi_2 \phi_2^\dagger \phi_2 + \text{H.c.} \quad (1)$$

Then we impose a Peccei-Quinn type symmetry, Q_1 , to eliminate the non-hermitian quartic terms of dimension-4. However we shall allow the soft terms such as $-m_{12}^2 \phi_1^\dagger \phi_2$ to break the Q_1 symmetry. Beyond tree level, the λ_5 term as well as other non-hermitian quartic Higgs couplings, λ_6 and λ_7 will be induced as quantum corrections.

At tree level, since the only non-hermitian coupling is m_{12}^2 , the ground state is CP symmetric automatically as the relative phase between $\langle\phi_1\rangle$ and $\langle\phi_2\rangle$ is zero. One may think that, with the induction of non-hermitian quartic terms from the Higgs-boson loops, it might be possible to produce a CP violating ground state by fine-tuning. However, that is not the case because the induced quartic non-hermitian couplings will be proportional to m_{12}^2 and cannot be used to balance the tree level coupling, m_{12}^2 , no matter how much one tunes. Even worse, the sign of the leading contribution to λ_5 is negative. It was shown in Ref.[6] that to get a CP violating ground state in the two Higgs doublet model it is necessary that the induced λ_5 term is positive.

To induce a positive λ_5 for our purpose, we need to enlarge the particle content further. Here we choose to enlarge the leptonic sector by two additional right-handed neutrino, N_{1R} and N_{2R} , in addition to the usual lepton doublet L . The spectrum of the model now looks like:

$$\begin{array}{|c|c|c|c|c|c|} \hline & \phi_1 & \phi_2 & L & N_1 & N_2 \\ \hline Y & 1 & 1 & 1 & 0 & 0 \\ Q_1 & 2 & 0 & 1 & -1 & 1 \\ Q_2 & 0 & 0 & 1 & 1 & 1 \\ \hline \end{array}, \quad (2)$$

where Y is the hypercharge and Q_2 is lepton number symmetry, which is automatic as far as the dimension-4 couplings are concern. The relevant Yukawa interactions are

$$-\mathcal{L}_Y(N) = f_1 \bar{L} N_{1R} \phi_1 + f_2 \bar{L} N_{2R} \phi_2 + \text{H.c.} \quad (3)$$

We assume the global Q_1 symmetry on the hard (dimension-4) terms but we allows the soft terms to break the symmetry. They contain, in addition to the m_{12} term, the following three Majorana mass terms

$$-\mathcal{L}_{SB}(\text{dim-3}) = \mu_{12} N_1^T C N_2 + \mu_{11} N_1^T C N_1 + \mu_{22} N_2^T C N_2 + \text{H.c.} \quad (4)$$

It is important to note that a discrete symmetry,

$$Z_2 : N_1 \rightarrow -N_1, \quad \phi_1 \rightarrow -\phi_1, \quad (5)$$

is respected by all terms in the Lagrangian except by the terms m_{12}^2 and μ_{12} . Therefore it is natural to fine-tune these two couplings m_{12}^2 and μ_{12} to be small. Also, one should keep in mind that the Majorana masses break the lepton number symmetry Q_2 softly.

Note that μ_{11}, μ_{22} mass terms are $SU(2) \times U(1)$ invariant. Therefore, their values can in principle be much larger than the $SU(2)$ breaking scale.

Before we get into the discussion of CP symmetry breaking, it is also interesting to note that if one set the coupling m_{12} to zero, it will get divergent contribution induced by the μ_{12} term, however the divergence is only logarithmic with coefficient proportional to $\mu_{12}(\mu_{11} + \mu_{22})f_1f_2$. In addition, since the couplings λ_6 and λ_7 are also forbidden by the Z_2 discrete symmetry, their induced values will be proportional to μ_{12} or m_{12} also. Therefore, by fine-tuning the parameters μ_{12} and m_{12} to be small one can make all the couplings which are forbidden by the Z_2 symmetry small (relative to the dimensional parameters μ_{11} and μ_{22}). Near this limit, one can find a CP violating ground state.

To break CP symmetry spontaneously, the loop-induced λ_5 must have a positive sign. This can be achieved by diagrams in Fig. 1.

$$\lambda_5 = -\frac{f_{11}^2 f_{22}^2}{16\pi^2} \frac{\mu_{11}\mu_{22}}{\mu_{11}^2 - \mu_{22}^2} \log \frac{\mu_{11}^2}{\mu_{22}^2}. \quad (6)$$

The positive sign of λ_5 can always be achieved when μ_{11} and μ_{22} are of opposite signs.

The minimization of the potential for the most general couplings has already been done in Ref.[6]. Parametrizing the vacuum expectation value (VEV) as $\langle\phi_1\rangle = v_1 e^{i\delta}/\sqrt{2}$ and $\langle\phi_2\rangle = v_2/\sqrt{2}$, we obtain

$$\cos \delta = \frac{2m_{12}^2 - \lambda_6 v_1^2 - \lambda_7 v_2^2}{4\lambda_5 v_1 v_2}. \quad (7)$$

(The condition that the vacuum preserves $U(1)_{EM}$ is also analyzed in Ref.[6]). First of all, since $\lambda_6 v_1^2$ and $\lambda_7 v_2^2$ are simultaneously one-loop induced and Z_2 breaking, they are naturally small compared with m_{12}^2 , which is a tree level Z_2 breaking term, and therefore negligible. To have a significant CP violating phase δ we need m_{12}^2 to be of the same order as $\lambda_5 v_1 v_2$. This requires the fine-tuning of the tree level coupling m_{12} . The fine-tuning is technically natural if we simultaneously fine-tune both m_{12} and μ_{12} because they are the only two tree-level terms forbidden by the Z_2 symmetry. Therefore, one has arrived at a model in which the CP symmetry is spontaneously broken by the radiative correction. (In contrast, it is easy to show that the supersymmetric models such as the one proposed by Maekawa[6] is not technically natural).

Since we have chosen to extend the lepton sector of the Standard Model, we shall next discuss the structure of the neutrino masses in the model. Consider the one-generation

case. The Majorana mass matrix of the model can be written as

$$\begin{array}{c} \begin{array}{|c|c|c|c|} \hline & \nu^C & N_{1R} & N_{2R} \\ \hline \nu^C & 0 & f_1 \frac{v_1}{2\sqrt{2}} & f_2 \frac{v_2}{2\sqrt{2}} \\ N_{1R} & f_1 \frac{v_1}{2\sqrt{2}} & \mu_{11} & \frac{1}{2}\mu_{12} \\ N_{2R} & f_2 \frac{v_2}{2\sqrt{2}} & \frac{1}{2}\mu_{12} & \mu_{22} \\ \hline \end{array} \end{array} . \quad (8)$$

where ν^C is from the usual left-handed neutrino. The neutrino spectrum is nothing but the usually see-saw spectrum of one very light and two very heavy Majorana particles. This is especially true if one assumes that the singlet masses, $|\mu_{11}|$, $|\mu_{22}|$, are much larger than $SU(2)$ breaking scale (while $|\mu_{12}|$, $\ll |\mu_{11}|$, $|\mu_{22}|$, is fine-tuned be small). Increase the number of generation by adding an index to ν^C is going to simply increase the number of light Majorana particles.

Next, we deal with the problem of a potentially light pseudoscalar boson A with a mass $m_A = \sqrt{2\lambda_5}v \sin \delta$ in our model where $v = \sqrt{v_1^2 + v_2^2}$. The value of $\lambda_5^{1/2}$ in Eq. (6) can be naturally as large as 0.1. m_A is easily around 30 GeV. The masses of the other scalar bosons are usually much larger. The potential limit on the mass of a pseudoscalar Higgs boson comes from LEP experiments. However, in all the analyses[8], the pseudoscalar bosons are assumed to be produced by the decay of a scalar boson H . For the case when the scalar boson is very heavy (such as $m_H > m_Z$), no limit on m_A has been extracted yet. One may try to obtain a limit on the pseudoscalar boson by considering the emission $Z \rightarrow Z^* AA \rightarrow l^+ l^- AA$ [11]; however the branching ratio is about 10^{-8} , too small for the present LEP data unless the $ZZAA$ gauge vertex is very large for some peculiar reason which does not happen in this model. A pseudoscalar Higgs boson lighter than a b quark can be ruled out by $b \rightarrow sA$. [12]

Note that in the limit that the Higgs potential has a custodial $SU(2) \times SU(2)$ symmetry, the pseudoscalar boson mass is the same as the charged Higgs boson mass at the tree level[13]. Of course, in our case, not only the Higgs potential contains a parameter which does not respect the custodial symmetry. The CP violating ground state we obtained also breaks the custodial symmetry. These breakings can contribute to the ρ parameter at the one loop level, however the resulting constraint[13] are not significant numerically for the model considered here.

Finally, we shall make a short discussion of the CP phenomenology. The details of

the CP phenomenology depend on how the Higgs doublets are coupled to the quarks[14, 15, 16]. Since we have only touched upon the leptonic sector to produce radiative CP violation so far, there are some arbitrariness in deciding how the quarks are coupled. Basically, these doublets can couple to quarks in two different ways. The first way is to couple one of the Higgs doublets to the up-type quarks u_R and the other one to the down-type quarks d_R . This is the way chosen in Peccei-Quinn mechanism[9]. The second way is to couple both types of quarks u_R, d_R to one and same doublet. We shall only discuss the first option here even though the second option may also be interesting.

The leading mechanism of CP violation is through the neutral Higgs boson exchange. Since the tree level couplings of the neutral Higgs bosons are flavor conserving, the leading contribution to the CP violating ϵ parameter in the kaon system is through the two-loop diagrams[6, 16]. The mechanism also tends to give large contribution to the neutron electric dipole moment, d_n . While it is generally believed that the neutral Higgs boson exchange alone is not enough to account for all the known CP phenomenology [15], there are however some claims in the literature Ref.[16] that, by properly adjusting parameters, it is possible to produce large enough ϵ with small enough d_n in some models of neutral Higgs mediated CP violation. We shall not get deeply into this complicated and detailed phenomenological issue here because it is not really directly connected to the main issue we wish to illuminate. If it is indeed the case that some tree level flavor changing neutral currents are needed to produce large enough ϵ , it can easily be accommodated in this model by a small extension of the quark sector such as adding a vectorial down quark[17] which appears in E_6 -type grand unified theories. It is also known that the CP violating ϵ' of the kaon decay and CP violating parameters in hyperon decays are both negligible in this type of model. Detail analysis of this and various other possibilities will be presented elsewhere.

The strong CP problem in models with soft breaking of Peccei-Quinn symmetry is discussed in Ref.[18].

Note that even if one does not impose CP symmetry on the Lagrangian the Higgs sector alone is automatically CP conserving at tree level. Therefore, even in models with other source of CP violation (such as the Kobayashi-Maskawa mechanism[1]), quantum effects can produce a new independent source of the CP violation. Of course, in that case,

one can no longer claim that the CP violation is a quantum mechanical effect.

Finally let us address on the natural scale for the singlet. Taking the simplifying assumption that $v_1 \sim v_2 \sim v$, $f_1 \sim f_2 \sim f$ and $\mu_{11} \sim \mu_{22} \sim \mu$, we can correlate the pseudoscalar mass $m_A \sim f^2 v / (4\pi)$ and the light neutrino mass $m_\nu \sim f^2 v^2 / \mu$ in the relation $\mu \sim 4\pi v (m_A / m_\nu)$. For $m_\nu \sim 10$ eV, one needs $\mu \sim 10^{13}$ GeV. In a grand unified theory, the singlet scale μ is presumably related to the grand unified scale or an intermediate scale. Therefore having a high singlet scale is not a serious problem.

To conclude, we have shown that if one allows fine-tuning which is technically natural, it is relatively easy to construct models in which the tree level vacuum is CP invariant while the loop-corrected potential produces CP nonconserving vacuum. The basic ingredient is to impose enough symmetry (the global Q_1 symmetry in our example) on the higher dimensional terms such that the tree level potential has only one soft, non-hermitian, symmetry breaking term. Then the loop-induced higher order term can produce the desired CP nonconserving vacuum through fine-tuning. To make the fine-tuning technically natural, one then has to find a smaller symmetry (Z_2 in our case) which can forbid some of the soft terms while allowing the others. Since the softer terms typically have smaller discrete symmetry than the hard terms, such symmetry is not too hard to find either in general. The example we provided in this paper is not only simpler than the supersymmetric models in the literature, it is also more appealing because the necessary fine-tuning in our case is technically natural.

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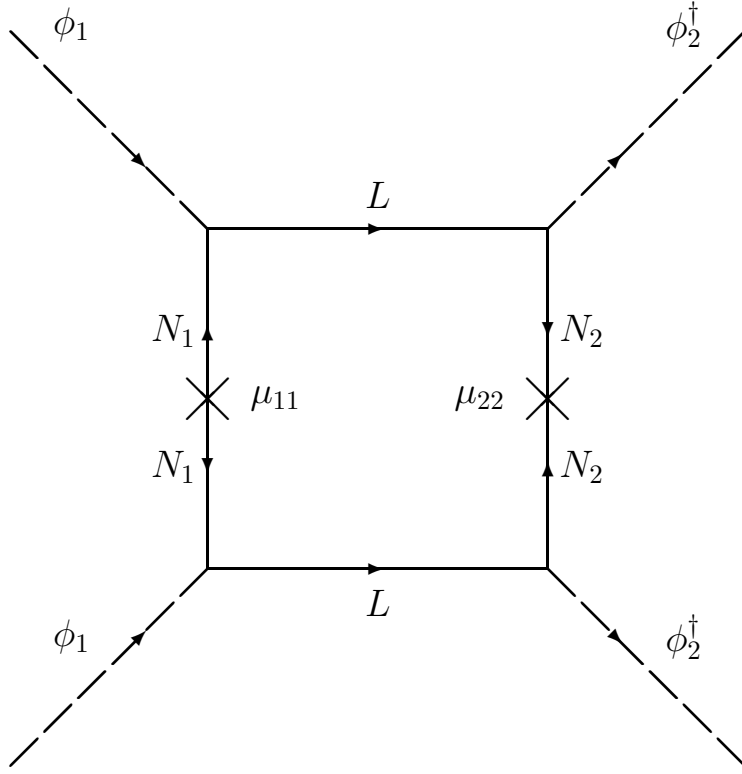


Fig. 1 Feynman diagram for the induced vertex $\lambda_5(\phi_1\phi_2^\dagger)^2$ via the fermion loop.